

# DynIBEX: a Differential Constraint Library for Studying Dynamical Systems

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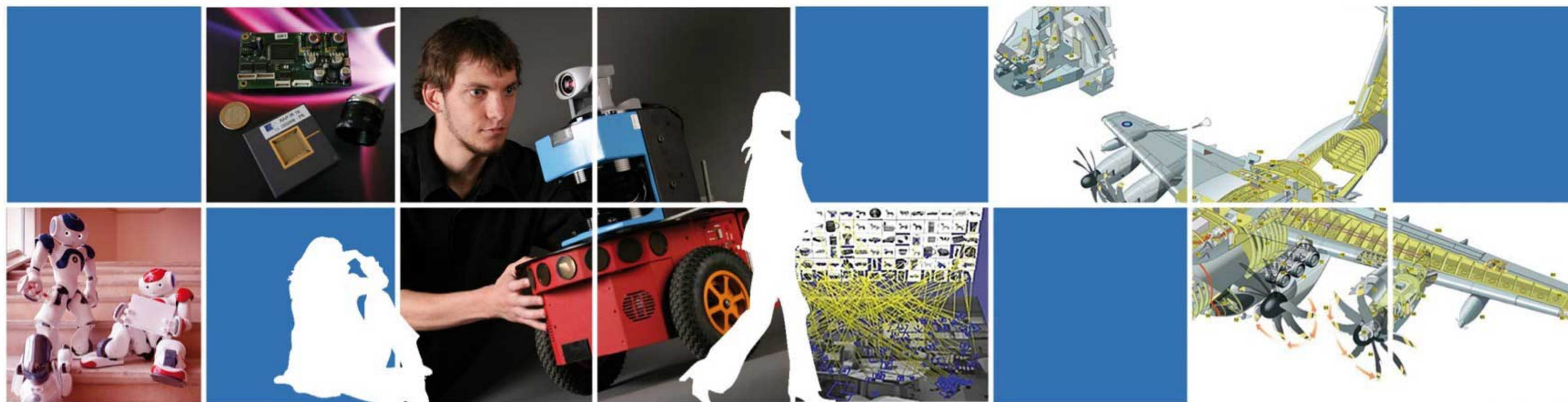
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## DynIBEX: a Differential Constraint Library for Studying Dynamical Systems

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**A free open-source library combining validated numerical integration methods with a constraint programming approach, as a plugin of IBEX library with:**

- validated integration methods based on Runge-Kutta for Ordinary Differential Equations (ODEs) and Differential Algebraic Equations (DAEs)
- operators for satisfaction problems on dynamical systems

<http://perso.ensta-paristech.fr/~chapoutot/dynibex/>

### 1-Differential Equation Supported

**Parametric and Constrained :**

$$\begin{cases} \dot{y} = F(t, y, x, p, u) \\ 0 = G(t, y, x, p, u) \\ 0 = H(t, y, p, u) \end{cases}$$

with  $y(0) \in [y_0]$ ,  $x(0) \in [x_0]$ ,  $p \in [p]$ ,  $u \in [u]$  and  $t \in [0, t_{end}]$ .

With:

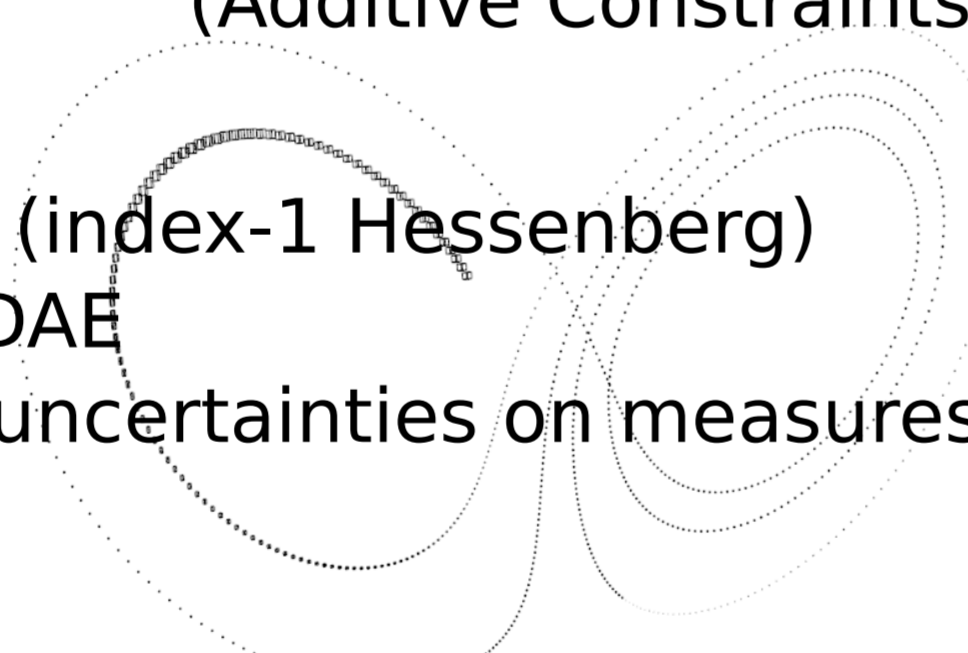
$F : R \times R^n \times R^m \times R^r \times R^s \mapsto R^n$  (Differential Part)

$G : R \times R^n \times R^m \times R^r \times R^s \mapsto R^m$  (DAE part)

$H : R \times R^n \times R^r \times R^s \mapsto R^c$  (Additive Constraints)

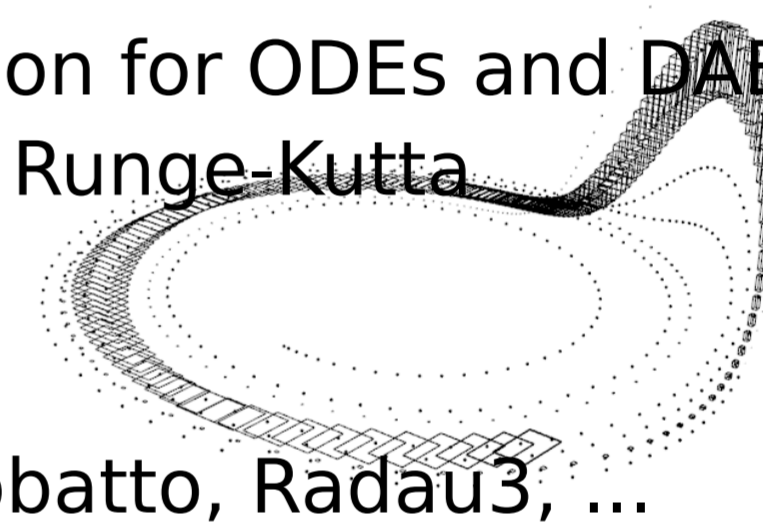
Note that:

- $m=0$ : ODE, otherwise DAE (index-1 Hessenberg)
- $c>0$ : Constrained ODE or DAE
- $r>0$ : Interval Parameters (uncertainties on measures, modeling, parameters)
- $s>0$ : Embedded control



### 3-Validated Simulation with Runge-Kutta

- Proof of existence and uniqueness of solution for ODEs and DAEs
  - Local truncation error computation for any Runge-Kutta method (implicit or explicit)
  - Combined with contractors (HC4)
- => Validated Runge-Kutta: Heun, RK4, Lobatto, Radau3, ...



### 4- Verification of temporal constraints

Stayed in  $\mathcal{A}$  (until  $\tau$ ) :  $y(t) \subset \text{Int}(\mathcal{A}), \forall t (\forall t < \tau)$

Included in  $\mathcal{A}$  at  $\tau$  :  $y(\tau) \subset \text{Int}(\mathcal{A})$

Has crossed  $\mathcal{A}$  (before  $\tau$ ) :  $\exists t, y(t) \cap \square \mathcal{A} \neq \emptyset (\exists t < \tau)$

Gone out  $\mathcal{A}$  (before  $\tau$ ) :  $\exists t, y(t) \cap \square \mathcal{A} = \emptyset (\exists t < \tau)$

Has reached  $\mathcal{A}$  :  $y(t_{end}) \cap \square \mathcal{A} \neq \emptyset$

Finished in  $\mathcal{A}$  :  $y(t_{end}) \subset \text{Int}(\mathcal{A})$

### 5- Example of simulation and constraints

Model of an Hovercraft:

$$\begin{cases} \dot{u} = vr + \frac{1}{m} \left( F_u - \mu_{sec} N f(u) \frac{u}{\sqrt{u^2 + v^2}} - \frac{1}{2} \rho C_d S_u \sqrt{u^2 + v^2} u - D_{11} u \right) \\ \dot{v} = -ur + \frac{1}{m} \left( \frac{1}{4A} C_L S_v F_v \delta - \mu_{sec} N f(v) \frac{v}{\sqrt{u^2 + v^2}} - \frac{1}{2} \rho C_d S_v \sqrt{u^2 + v^2} v - D_{22} v \right) \\ \dot{r} = \frac{1}{I_z} \left( -L \frac{1}{4A} C_L S_f F_r \delta - D_{33} r \right) \\ \dot{x} = \cos(\psi) u - \sin(\psi) v \\ \dot{y} = \sin(\psi) u + \cos(\psi) v \\ \dot{\psi} = r \end{cases}$$

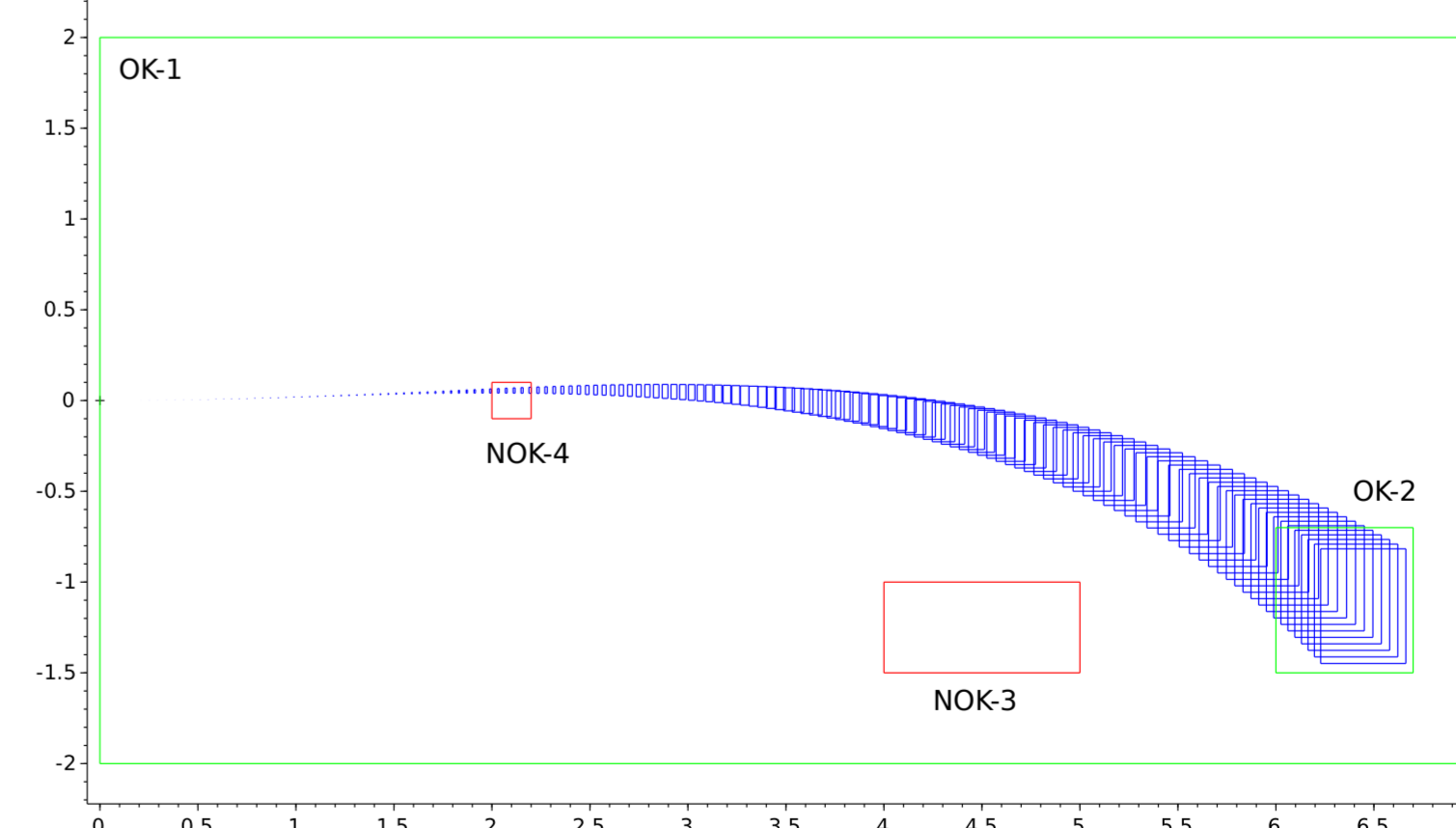
**OK-1:** Stayed in safe zone

**OK-2:** Finished in goal

**NOK-3:** Avoid obstacle

**NOK-4:** Not in forbidden zone at  $\tau$

=> **Design, control synthesis, parameter identification, model simplification, ...**



### 2-Existing tools in IBEX

**A library for interval constraint programming:**

- Interval and affine arithmetic
- Function evaluation
- Contractor programming (HC4, Newton, ...)
- Bisection (Branch & Prune)
- Set operators (inclusion, intersection, ...)

### DynIBEX applications

Controller synthesis (PID, switched systems), design, viability kernel, predictive control, sensitivity analysis

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